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Why do we have an interbank money market?

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Abstract

The interbank money market plays a key role in the execution of monetary policy. Hence, it is important to know the functioning of this market and the determinants of the interbank money market rate. In this paper, we develop an interbank money market model with a heterogeneous banking sector. We show that besides for balancing daily liquidity fluctuations banks participate in the interbank market because they have different marginal costs of obtaining funds from the central bank. In the euro area, which we refer to, these cost differences occur because banks have different marginal cost of collateral which they need to hold to obtain funds from the central bank. Banks with relatively low marginal costs act as intermediaries between the central bank and banks with relatively high marginal costs. The necessary positive spread between the interbank market rate and the central bank rate is determined by transaction costs and credit risk in the interbank market, total liquidity needs of the banking sector, costs of obtaining funds from the central bank, and the distribution of the latter across banks.

Zusammenfassung

Der Interbankenmarkt hat eine zentrale Bedeutung für die Durchführung der Geldpolitik. Daher ist es wichtig, die Funktionsweise dieses Marktes und die Determinanten des Interbankenmarktzinssatzes zu verstehen. In diesem Beitrag entwickeln wir ein Modell mit einem heterogenen Geschäftsbankensektor. Traditionell wird die Existenz des Interbankenmarktes damit erklärt, dass Banken einen Markt benötigen, um tägliche Liquiditätsschwankungen auszugleichen. Wir zeigen, dass ein Interbankenmarkt auch dann entsteht, wenn sich die Grenzkosten der Kreditbeschaffung bei der Zentralbank zwischen den Geschäftsbanken unterscheiden. Im Euroraum, auf den sich dieser Beitrag bezieht, sind diese Kostenunterschiede auf unterschiedliche Kosten der Besicherung von Zentralbankkrediten zurückzuführen. Dabei fungieren Banken mit relativ niedrigen Grenzkosten als Intermediäre zwischen der Zentralbank und Banken mit relativ hohen Grenzkosten. Die dafür notwendige positive Differenz zwischen dem Interbankenmarktzins und dem Zentralbankzins wird durch die Transaktionskosten und das Kreditrisiko im Interbankenmarkt, den aggregierten Liquiditätsbedarf des Bankensektors, die Kosten der Kreditbeschaffung bei der Zentralbank sowie deren Verteilung über die Banken bestimmt.

JEL classification: E43, E52, E58, G21

Keywords: interbank money market, European Central Bank, monetary policy instruments

1 Introduction

The interbank money market, and here especially the market for unsecured overnight loans, plays a crucial role in the conduct of monetary policy. It is the starting point for the transmission mechanism of monetary policy impulses, and in most industrialized countries, the rate on these overnight loans is the central bank's operating target. Hence, for the conduct of monetary policy it is important to know the functioning of this market and the determinants of the interbank money market rate.

The interbank money market reallocates the liquidity originally supplied by the central bank. One reason for this reallocation is the offset of anticipated and non-anticipated daily liquidity imbalances. Furthermore, banks are motivated to take part in the interbank market for speculative purposes. This paper derives an additional reason for banks to participate in the interbank money market: a heterogenous banking sector. Banks have different marginal cost of obtaining funds from the central bank. In the euro area, these cost differences occur because marginal opportunity cost of collateral, which banks need to hold to obtain funds from the central bank, vary across countries within the euro area (Hämäläinen 2000). Developing an interbank market model capturing this aspect, we show that in this case intermediation occurs. Banks with relatively low marginal cost act as intermediaries between the central bank and credit institutions with relatively high marginal cost.

This intermediation has important ramifications for the conduct of monetary policy as the following example shows. The main refinancing operations (MROs) are the Eurosystem's key instrument to provide liquidity to the banking sector in the euro area. The MROs are credit transactions with a two week maturity which are executed weekly either through a fixed or variable rate tender.[1] In the past, several MROs were characterized by underbidding behaviour[2] which led to a sizeable increase in the interbank money market rate. This underbidding behaviour occurred when banks expected the central bank to lower interest rates within the maturity of the respective MRO. The extremely low demand for funds at the central bank

[1] Some information on the MROs are given in section 2. For a detailed description of the MROs and the other monetary instruments of the Eurosystem see, for example, ECB 2002c. However, it should be noted that the Eurosystem will change its operational framework. For details concerning the intended alterations see ECB 2003a.

[2] Underbidding behaviour is characterized by an assessment of actual liquidity needs of the banking sector in the euro area, considering smooth provisions of required reserves (see ECB 2002b for details).

can be attributed to speculation on behalf of the banks and to a reduced incentive to intermediate (for a theoretical analysis see Neyer 2003). The Eurosystem could have prevented the strong increase in the interbank market rate by providing the necessary additional liquidity. But usually the Eurosystem did not want to offset the liquidity deficits in order to drive home the point that underbidding behaviour is a non-profit-making strategy for the banks (see, for example, ECB 2001a p. 16). This kind of "education" may work to prevent banks from speculating, but obviously, it does not help to prevent a reduced incentive to intermediate. Therefore, if intermediation plays an important role in the interbank market, this kind of "education" will be fruitless.

Obviously, a bank will only act as an intermediary if there is a positive spread between the interbank market rate and the central bank rate. Empirical studies confirm this positive spread for the euro area (see Nyborg, Bindseil, and Strebulaev 2002; Ejerskov, Moss, and Stracca 2003; Ayuso and Repullo 2003). However, whereas Ayuso and Repullo take the positive spread as a support for their assumption of an asymmetric objective function of the Eurosystem, we argue that the positive spread is due to a heterogenous banking sector. Banks face different marginal cost of obtaining funds from the central bank. With the help of our interbank market model, we derive the following determinants of the spread between the interbank market rate and the central bank rate: transaction costs and credit risk in the interbank market, total liquidity needs of the banking sector, collateral's opportunity costs, and the distribution of the latter across banks.

The bulk of related literature analyzes the U.S. federal funds market. Developing a model in which individual banks compare the liquidity benefit of excess reserves with the federal funds rate, Ho and Saunders (1985) derive different federal funds demand functions and provide several explanations for specific features of the federal funds market. Clouse and Dow (2002) model the reserve management of a representative bank as a dynamic programming problem capturing main institutional features of the federal funds market to discuss the effects of various changes to the operating environment and monetary policy instruments. A huge part of the literature dealing with the federal funds market analyzes why the federal funds rate fails to follow a martingale within the reserve maintenance period, i.e. why banks obviously do not regard reserves held on different days of the maintenance period as perfect substitutes (see, for example, Hamilton 1996; Clouse and Dow 1999; Furfine 2000; Bartolini, Bertola, and Prati 2001, 2002a).

However, Bartolini, Bertola and Prati (2002b) demonstrate that explanations for key behavioural features of the U.S. federal funds rate cannot be used for explaining the behaviour of the short term interest rates in other countries, but that country specific central banks' operating procedures play a crucial role in determining this interbank rate.[3] This is reflected by a number of papers considering typical features of the Eurosystem's operational framework. Capturing main characteristics of this framework, an extensive number of papers deals with the causes and consequences of the banks' under- and overbidding behaviour in the MROs (see, for example, Ayuso and Repullo 2001, 2003; Bindseil 2002; Ewerhart 2002, Ewerhart et al 2003, Nautz and Oechssler 2003, and Neyer 2003). Pérez-Quirós and Rodríguez-Mendizábal (2001) construct a model where the interest rates of the Eurosystem's two standing facilities play a crucial role in determining the behaviour of the interbank market rate within a reserve maintenance period. Välimäki (2001) presents an interbank market model to analyze the performance of alternative fixed rate tender procedures.

Our paper contributes to the literature by modelling an interbank money market with a heterogenous banking sector. Banks differ in marginal cost of obtaining funds from the central bank because they have different marginal opportunity cost of holding collateral which implies that intermediation occurs. Our analysis allows us to offer an explanation for the observed positive spread between the interbank market rate and the central bank rate in the euro area.

The remainder of this paper is structured as follows. Section 2 gives some institutional background information on the interbank money market in the euro area and presents some stylized facts about the spread between interbank market and central bank rates. Section 3 models an interbank money market with a heterogenous banking sector, and section 4 summarizes the paper.

2 The Interbank Money Market in the Euro Area

Institutional Background

In the euro area, liquidity needs of the banking sector mainly arise from two factors: the so-called autonomous factors as banknotes in circulation and government de-

[3]For a detailed comparison of the Eurosystem's and the Federal Reserve System's operational frameworks, for example, see Ruckriegel and Seitz 2002. Bartolini and Prati (2003), also comparing the two central banks, focus on the different approaches to the execution of the monetary policy.

posits with the Eurosystem and minimum reserve requirements. The Eurosystem's minimum reserve system requires credit institutions to hold a fixed amount of compulsory deposits on the accounts with the Eurosystem. For fulfilling these reserve requirements, averaging provisions are allowed over a one-month reserve maintenance period.[4] The bulk of these liquidity needs (about 74 %) are satisfied by the Eurosystem through the MROs. About 26 % of the liquidity needs are met through longer-term refinancing operations, less than 1 % through fine-tuning operations. Finally, residual liquidity needs (only about 0.4 %) are balanced by the banks' recourse to the marginal lending facilities.[5] The key instrument of the Eurosystem to provide liquidity to the banking sector in the euro area, the MROs, are credit transactions with a two-week maturity which are executed weekly. They are conducted either as a fixed rate or a variable rate tender.[6] From the launch of the euro in January 1999 until June 2000, tenders were conducted exclusively as fixed rate tenders. Since then, only variable rate tenders with a minimum bid rate have been used. For our analysis it is important that the MROs have to be based on adequate collateral.[7] Although differences in the financial structure across Member States of the EMU have been considered when defining the list of eligible assets, marginal costs of collateral vary across countries within the euro area (Hämäläinen 2000).

The liquidity supplied by the Eurosystem is reallocated via the interbank money market. This market can be divided into the cash market, the market for short-term securities and the market for derivatives.[8]

The cash market consists of the unsecured market, the repo market and the foreign exchange swap market. In the unsecured market, activity is concentrated on the overnight maturity segment. The reference rate in this segment is the Eonia (Euro Overnight Index Average). It is a market index computed as the weighted average of overnight unsecured lending transactions undertaken by a representative panel of banks. The same panel banks contributing to Eonia also quote for Euribor (Euro

[4]For a detailed description of the current Eurosystem's minimum reserve system see, for example, ECB (2002c). However, the Eurosystem will change the timing of the reserve maintenance period. For details concerning the intended alterations see ECB (2003a).

[5]For a detailed description of the demand for and the supply of liquidity in the euro area see ECB (2002b). The data given in this paragraph are averages over the period from January 1999 until December 2001. Source: ECB (2002b).

[6]The Eurosystem will change its operational framework. Inter alia, it will shorten the maturity of the MROs from two weeks to one week. For details concerning the intended alterations see ECB 2003a.

[7]Eligible assets have been defined by the Eurosystem. For details see ECB (2002c, p. 38-50).

[8]For more detailed information on the euro money market we refer the reader to ECB 2001b, 2002a, 2003b.

Interbank Offered Rate). Euribor is the rate at which euro interbank term deposits are offered by one prime bank to another prime bank. This is the reference rate for maturities of one, two and three weeks and for twelve maturities from one to twelve months.[9]

The market for short term securities includes government securities (Treasury bills) and private securities (mainly commercial paper and bank certificates of deposits). In the market for derivatives, typically interest rate swaps and futures are traded.

The purpose of our paper is to show that due to cost differences between banks in obtaining funds from the central bank intermediation occurs. Looking at the euro area, one obtains the most obvious hint of intermediation when considering that only a fraction of all banks actually takes part in the MROs.[10] A further hint would be an on average positive spread between the interbank market rate and the rate banks have to pay at the central bank. The following empirical analysis shows that the spread is significantly positive.

Money Market Rate and ECB Rate: Test of the spread

The spread between the interbank market rate and the rate banks have to pay at the central bank has been examined in a number of recent publications (for example Ayuso and Repullo 2003; Ejerskov, Moss, and Stracca 2003; Nyborg, Bindseil, and Strebulaev 2002). While the positiveness of the spread is not explicitly tested in the latter two publications, Ayuso and Repullo find a significantly positive spread. Our test differs from Ayuso and Repullo's in the interest rates used to approximate the interbank market rate and the central bank rate as well as in the samples used for the analysis. We started our analysis by comparing the key central bank rate in the euro area, i.e. the fixed rate applied to the fixed rate tenders and the minimum bid rate applied to the variable rate tenders, with the key interbank money market rate, i.e. the Eonia. Our sample of daily observations ranges from 4 January 1999 to 23 September 2003, resulting in 1232 observations. The data are drawn from

[9] For more information on these reference rates see www.euribor.org.

[10] At the end of 2000 for example, 2,542 credit institutions in the euro area fulfilled the criteria for participating in the main refinancing operations, but in 1999 and 2000 the total number of institutions which actually took part in these operations fluctuated between 400 and 600 (ECB 2001c, p. 63). Also in 2001 and 2002 the number of banks taking part in the MROs was relatively small: it fluctuated between 175 and 658, on average 357 banks took part in the MROs.

ECB sources[11]. Figure 1 shows the Eonia and the respective MRO-rate. The spread between those series is displayed in figure 2.

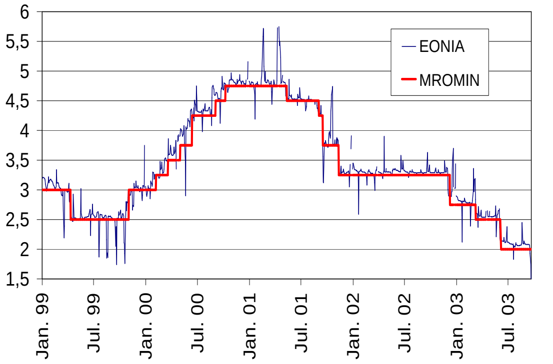

Figure 1: Eonia and MRO-rate, i.e. the rate applied to the fixed rate tenders and the minimum bid rate of the variable rate tenders.

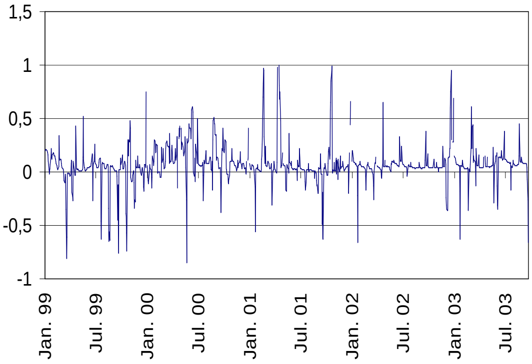

Figure 2: Spread between the Eonia and the MRO-rate, i.e. the rate applied to the fixed rate tenders and the minimum bid rate of the variable rate tenders.

Obviously, the Eonia has usually been close to the MRO-rate, except for some infrequent spikes which coincide with some special episodes in the sample period. These are:

- Underbidding episodes in February, April and October 2001, December 2002 and March 2003. More underbidding episodes occurred (April 1999, November 2001 and June 2003), but they did not lead to tight conditions in the interbank market and thus had no significant effect on the Eonia.
- Anomalous allotment on 18 September 2001, the week following the terrorist attack in the US.
- End of year and cash changeover effects.
- End of reserve maintenance periods effects. The allowance of averaging provisions of required reserves over a reserve maintenance period typically results in a strong activity in the interbank market on the last days of the maintenance period and a relatively high corresponding change in the Eonia on that days.
- Periods between the governing council's announcement of an interest rate change and its implementation.

[11] Time series are available on the ECB website www.ecb.int.

Testing for a positive spread, we excluded these periods from the sample because our goal is to test the positiveness of the spreads under "normal" conditions. Furthermore, we restrict the sample to the days of settlement of the MROs so that the results of the tests are comparable to the following tests of the Euribor spread. The first column of table 1 reports the one-sided test of the null hypothesis of a non-positive spread against the alternative of a positive spread between the Eonia and the MRO-rate. The average spread was 10.5 basis points during the fixed tender period and fell to 6.2 basis points during the variable tender period. The null hypothesis of a non positive spread can be rejected on a confidence level of 1%.

However, this test involves two potential biases that might affect the spread. First, the MROs have a two-week maturity which implies that the MRO-rate has a positive term premium when compared to the Eonia which refers to overnight transactions. This should bias the spread downwards. Second, differences in credit risk may bias the spread upwards since the MROs are collateralized while the Eonia refers to unsecured interbank market transactions.[12] In order to reduce the first bias, we used the two-week Euribor for testing whether the spread is positive. The two-week Euribor has the same maturity as the MROs, thus the term premiums should be equal. Due to the fact that the two-week Euribor is available only since 15 October 2001, we also employed the one-week Euribor, which is available since January 1999, giving a much larger sample while the difference in maturity is only one week. The data on the one-week and two-week Euribor is available on www.euribor.org. The second bias should generally be small, since the Eonia and the Euribor are only offered to banks of first class credit standing. Additionally, we did not compare the respective Euribor with the minimum bid rate but with the weighted average rate during the variable rate tender period. The reason is that the latter is the more appropriate rate when comparing the actual costs of refinancing in the interbank market with those at the central bank. Figures 3 and 4 show the one-week and two-week Euribor spreads.

[12] Concerning a discussion of these two potential biases see also Ayuso and Repullo (2003).

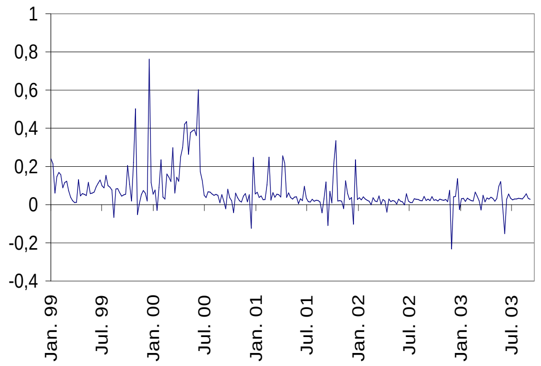

Figure 3: Spread between one-week Euribor and the MRO-rate, i.e. the rate applied to the fixed rate tenders and the weighted average rate of the variable rate tenders.

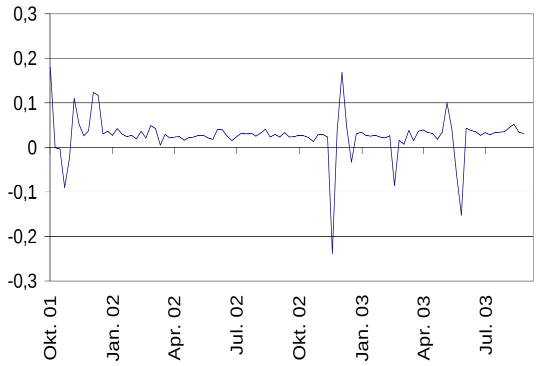

Figure 4: Spread between two-week Euribor and the MRO-rate, i.e. the rate applied to the fixed rate tenders and the weighted average rate of the variable rate tenders.

For testing the Euribor spreads we restrict the sample to the days of settlement of the MROs[13] and additionally exclude the same special episodes as for the test of the Eonia spread. The second and third column of table 1 report the one-sided tests of the null hypothesis of a non-positive spread against the alternative of a positive spread. The average one-week Euribor spread was 13.1 basis points during the fixed rate tender period and fell to 3.1 basis points during the variable rate tender period. The average two-week Euribor spread was 1.8 basis points. For all cases the null hypothesis of a non positive spread can be rejected on a confidence level of 1%.

	Eonia			1-week Euribor			2-week Euribor
	FT	VT	F+V	FT	VT	F+V	VT
Mean	0.105	0.062	0.076	0.131	0.031	0.064	0.018
(t-stat)	(6.807)	(11.131)	(11.864)	(8.901)	(8.042)	(9.885)	(3.647)
[p-value]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
N	61	127	188	61	125	186	74

Table 1: Tests using days of settlement, special events exluded. The interest rate data in the first block correspond to the Eonia spread while the second and third block show the one-week and two-week Euribor spread. Tests are reported for the fixed rate tender period (FT), the variable rate tender period (VT) and for both periods combined (F+V). Two-week Euribor data is only available for part of the variable rate tender period. Each column reports the sample mean, its t-statistic, the p-value of the one-sided test of the null hypothesis mean ≤ 0 against the alternative mean > 0 and the sample size N. During the variable rate tender period the Eonia is compared with the minimum bid rate while the respective Euribor is compared with the weighted average rate.

[13] We restrict the sample to the days of settlement of the MROs because using daily data for the Euribor would imply that the maturity of the respective interbank term deposits would be longer than the maturity of the respective MRO.

To check for robustness of the spreads, we also tested the Eonia spread and the two Euribor spreads using daily data, with and without the exclusion of special events. Results are presented in the appendix. We can also reject the null hypothesis of a zero spread for all cases on a 1% level.

The next section develops an interbank market model explaining this positive spread between interbank market and central bank rates to be due to cost differences between banks of obtaining funds from the central bank.

3 A Simple Model of an Interbank Money Market

Liquidity Costs and Optimization

We consider a continuum of measure one of risk-neutral, isolated, price taking banks. All banks have the same given liquidity needs summarized by the variable R.[14] To cover its liquidity needs, a single bank can borrow liquidity from the central bank or in the interbank market, where it can also place excess liquidity.

The amount of credit bank i borrows from the central bank at the given rate l is denoted with $K_i \geq 0$.[15] This credit transaction with the central bank has to be based on adequate collateral. We assume that rate of return considerations induce a strict hierarchy of a bank's assets,[16] and that assets which can serve as collateral have a relatively low rate of return. A reason may be specific criteria eligible assets have to fulfill. Consequently, the collateralization of central bank credits incurs increasing marginal costs. This opportunity cost of holding collateral is given by

$$Q_i = q_i K_i + f(K_i), \tag{1}$$

where $f(K_i) \geq 0$, $f(0) = 0$, $f' \geq 0$, $f'' > 0$, and $f'(R) < \infty$. The bank specific parameter $q_i \geq 0$ represents different levels of marginal opportunity costs between

[14] In our model, the interbank market function of balancing daily liquidity fluctuations could be considered by modelling liquidity needs R as a bank-specific random variable or by adding bank-specific shocks. However, this would make the analysis more complicated without changing the main result of this paper.

[15] We do not model explicitly the tender procedures by which credit transactions between the Eurosystem and banks in the euro area are executed. This would not change the main results of our paper but it would make the analysis much more complex. We focus on the main point for our analysis: in the euro area, a single bank can cover its liquidity needs either by borrowing from the central bank or in the interbank market where it can also place excess liquidity.

[16] This approach can be compared with the one by Blum and Hellwig (1995). They consider a bank with deposits and equity. The bank can put these funds into loans to firms, government bonds or reserves of high powered money. Blum and Hellwig assume that rate of return considerations induce a strict preference for loans over bonds and for bonds over reserves.

banks (functions, variables, and parameters not indexed by i are the same for each bank). This heterogeneity among banks is a key feature of our model.

In the interbank money market, a bank can demand credit or place excess liquidity. Bank i's net position in the interbank market is given by

$$B_i = R - K_i \lesseqgtr 0. \tag{2}$$

Trading in the interbank market, the bank faces transaction costs given by

$$Z_i = zh(B_i), \tag{3}$$

where $h(B_i) \geq 0$, $h(0) = 0$, $h'(B_i > 0) > 0$, $h'(B_i < 0) < 0$, $h'(0) = 0$, $h''(B_i) > 0$, $h'(R) < \infty$, and the parameter $z > 0$. Furthermore, we assume the cost function to be symmetric, i.e. $h(B_i) = h(-B_i)$. This approach of increasing marginal transaction costs can be compared with the common method of modelling the liquidity role of reserves, which posits that banks incur increasing costs when liquidity deviates from a target level (see, for example, Campbell 1987; Bartolini, Bertola, and Prati 2001). The convex form reflects increasing marginal costs of searching for banks with matching liquidity needs and those resulting from the need to split large transactions into many small ones to work around credit lines.

Defining l as the interest rate on the central bank credit, and e as the interbank money market rate, bank i's total liquidity costs are

$$C_i = \begin{cases} K_i l + B_i e + Q_i + Z_i & \text{if} \quad B_i \geq 0 \\ K_i l + B_i ep + Q_i + Z_i & \text{if} \quad B_i < 0. \end{cases} \tag{4}$$

If $B_i < 0$, the bank places excess liquidity in the interbank market at the rate e. Since credit transactions in the interbank market are uncollateralized, there is a credit risk which is captured by p, with $0 < p < 1$, denoting the given average probability of success of interbank credits. Bank i minimizes total liquidity costs by choosing the optimal level of K_i, subject to $K_i \geq 0$. The first order conditions are given by

$$l + q_i + f' = e + zh' \qquad \text{if} \quad 0 \leq K_i^{opt} \leq R, \tag{5}$$

and

$$l + q_i + f' - zh' = ep \qquad \text{if} \quad R < K_i^{opt}. \tag{6}$$

Equation (5) represents the first order condition of a bank covering its liquidity needs R in the interbank market and at the central bank ($0 < K_i^{opt} < R$), at the central bank only ($K_i^{opt} = R$) or in the interbank market only ($K_i^{opt} = 0$). In the former two cases, the marginal cost of central bank funds is equated to the marginal cost of funds borrowed in the interbank market. In the latter case, the marginal cost of covering R at the central bank is higher or equal to the marginal cost of borrowing R in the interbank market. Equation (6) shows the first order condition of a bank which borrows more reserves than R from the central bank. In this case, the sum of the marginal cost of central bank funds and marginal transaction costs in the interbank market is equated to the marginal revenue in the interbank market.

Equations (5) and (6) implicitly give the optimal credit demand $K_{i,R<K_i}^{opt}(e,l,p,q_i,R)$ and $K_{i,0\leq K_i\leq R}^{opt}(e,l,q_i,R)$. Using the implicit function theorem we find that $K_{i,R<K_i}^{opt}(\cdot)$ and $K_{i,0\leq K_i\leq R}^{opt}(\cdot)$ are decreasing in q_i:

$$\frac{\partial K_{i,R<K_i}^{opt}(\cdot)}{\partial q_i} = \frac{\partial K_{i,0\leq K_i\leq R}^{opt}(\cdot)}{\partial q_i} = -\frac{1}{f'' + h''} < 0.$$

The credit risk $(1-p)$ introduces non-differentiable points in the optimal demand function $K_i^{opt}(\cdot)$. We find these points by evaluating equations (5) and (6) respectively at $\lim_{K_{i,R<K_i}^{opt}\to R} K_{i,R<K_i}^{opt}(\cdot) = K_{i,0\leq K_i\leq R}^{opt}(\cdot) = R$ and solving for q_i:

$$q_a = ep - l - f'(R) \tag{7}$$

$$q_b = e - l - f'(R). \tag{8}$$

A third non-differentiable point q_c is found by setting $K_{i,0\leq K_i\leq R}^{opt}(\cdot) = 0$ and solving equation (5) for q_i:

$$q_c = e - l + zh'(R) - f'(0). \tag{9}$$

Thus, the banks' optimal credit demand $K_i^{opt}(\cdot)$ is described by a piecewise-defined function:

$$K_i^{opt}(e,l,p,q_i,R) = \begin{cases} K_{i,R<K_i}^{opt}(\cdot) & \text{if } 0 \leq q_i < q_a \\ R & \text{if } q_a \leq q_i < q_b \\ K_{i,0\leq K_i\leq R}^{opt}(\cdot) & \text{if } q_b \leq q_i \leq q_c \\ 0 & \text{if } q_c < q_i. \end{cases}$$

Figure 1 illustrates this result. If $q_i \geq q_c$, bank i will cover its total liquidity needs in the interbank market, i.e. $K_i^{opt}(\cdot) = 0$ and $B_i = R$, because for any K_i the marginal

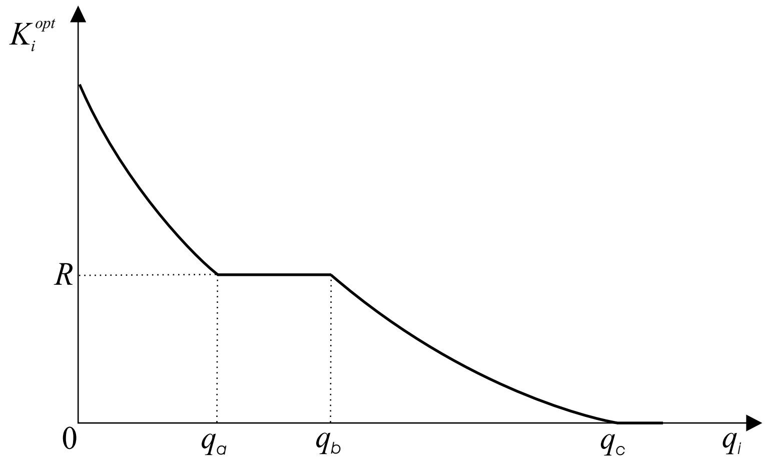

Figure 1: Optimal Credit Demand at the Central Bank

cost of the central bank credit exceeds the marginal cost of funds borrowed in the interbank market.

If $q_b < q_i < q_c$, bank i will partially cover its liquidity needs at the central bank and in the interbank market, i.e. $0 < K_i^{opt}(\cdot) < R$ and $0 < B_i < R$.

If $q_a \leq q_i \leq q_b$, bank i borrows exactly the amount from the central bank which covers its own liquidity needs, i.e. $K_i^{opt}(\cdot) = R$ and $B_i = 0$. It does not borrow more funds since expected interest earnings in the interbank market are not sufficient to cover its costs, and it does not borrow less reserves at the central bank because covering liquidity needs in the interbank market is more expensive. This perfectly inelastic behaviour of $K_i^{opt}(\cdot)$ between q_a and q_b is due to the credit risk in the interbank market $(1-p)$ which implies that interest cost per unit (e) and expected interest earnings per unit (ep) fall apart.

If $q_i < q_a$, the bank borrows more reserves from the central bank than it actually needs to cover its own requirements, i.e. $K_i^{opt}(\cdot) > R$. Its opportunity cost of holding collateral is relatively small, so that it is advantageous to borrow from the central bank to place liquidity in the interbank market.

Since $q_i \geq 0$, equations (7) to (8) reveal the obvious result that the interest rate in the interbank market must strictly be greater than the central bank rate $(e > l)$.

In figure 1, the slope of the $K_i^{opt}(\cdot)$ curve between 0 and q_a as well as between q_b and q_c has been chosen arbitrarily. Its exact shape depends on the form of the cost functions $f(K_i)$ and $h(B_i)$.

Equilibrium Interbank Market Rate

At the equilibrium interbank market rate e^*, liquidity supply in the interbank market equals liquidity demand. Therefore, assuming that q_i is distributed in the interval $[0, q_{max}]$ across banks according to the density function $g(q_i) = G'(q_i)$ with $G(0) = 0$, e^* is determined by

$$\begin{aligned} &\int_0^{q_a^*} (K^{opt}_{i,0\leq q_i<q_a^*}(\cdot) - R) g(q_i) dq_i \\ &= \int_{q_b^*}^{q_c^*} (R - K^{opt}_{i,q_b^*\leq q_i<q_c^*}(\cdot)) g(q_i) dq_i + \int_{q_c^*}^{q_{max}} R g(q_i) dq_i, \end{aligned} \tag{10}$$

where $q_a^* = e^* p - l - f'(R)$, $q_b^* = e^* - l - f'(R)$, and $q_c^* = e^* - l + zh'(R) - f'(0)$. The first line of equation (10) shows liquidity supply in the interbank market, the second liquidity demand of which the first integral represents demand by credit institutions covering partially their liquidity needs in the interbank market, whereas the second shows the demand of banks covering their total liquidity needs in that market. Equation (10) gives us the determinants of e^* and therefore of the spread $e^* - l$: transaction costs, the opportunity cost of holding collateral, the average credit risk in the interbank market, total liquidity needs R, and the distribution of q_i across banks. Applying the implicit function theorem and defining

$$\Delta \equiv \left(\int_0^{q_a^*} \frac{p g(q_i)}{f'' + zh''} dq_i + \int_{q_b^*}^{q_c^*} \frac{g(q_i)}{f'' + zh''} dq_i \right)^{-1} > 0.$$

we obtain:

$$\frac{\partial e^*}{\partial l} = \left(\int_0^{q_a^*} \frac{g(q_i)}{f'' + zh''} dq_i + \int_{q_b^*}^{q_c^*} \frac{g(q_i)}{f'' + zh''} dq_i \right) \Delta > 1, \tag{11}$$

$$\frac{\partial e^*}{\partial p} = -\int_0^{q_a^*} \frac{e^* g(q_i)}{f'' + zh''} dq_i \Delta < 0, \tag{12}$$

$$\frac{\partial e^*}{\partial R} = \left(G(q_a^*) + 1 - G(q_b^*) - \int_0^{q_a^*} \frac{zh'' g(q_i)}{f'' + zh''} dq_i - \int_{q_b^*}^{q_c^*} \frac{zh'' g(q_i)}{f'' + zh''} dq_i \right) \Delta > 0, \tag{13}$$

$$\frac{\partial e^*}{\partial z} = \left(-\int\limits_0^{q_a^*} \frac{h' g(q_i)}{f'' + zh''} dq_i - \int\limits_{q_b^*}^{q_c^*} \frac{h' g(q_i)}{f'' + zh''} dq_i \right) \Delta \gtreqless 0. \tag{14}$$

Equation (11) shows that an increase in the central bank rate leads to an even stronger increase in the interbank market rate. The reason is that a rising l leads to a likewise increase in the marginal cost of borrowing funds from the central bank (see equations (5) and (6)). Consequently, aggregate supply in the interbank market decreases whereas aggregate demand increases, implying the interbank market rate to rise to restore the market equilibrium. However, due to the credit risk $(1-p)$ in the interbank market a rising e does not lead to a likewise but smaller increase in the marginal revenue of the supplying banks (see equation (6)). Therefore, to restore optimality, $\partial e/\partial l > 1$.

Equation (12) reveals a positive relationship between the interbank market rate e and the credit risk in the interbank market $(1-p)$. The intuition is obvious: an increase in $(1-p)$ leads to a reduction in the marginal revenue of banks placing liquidity in the interbank market. Consequently, credit supply will decrease, leading to an increase in e.

There is also a positive relationship between the interbank market rate e and total liquidity needs R.[17] On the one hand, rising liquidity needs imply that the credit supply in the interbank market decreases since the supplying banks cover their additional needs by demanding more funds at the central bank *and* by reducing their supply in the interbank market (using equation (6) and employing the implicit function theorem reveals that $\partial K^{opt}_{i,R<K_i}(\cdot)/\partial R < 1$). On the other hand, an increase in R leads to an increase in the credit demand in the interbank market since the respective banks cover their additional liquidity needs by demanding more liquidity at the central bank *and* in the interbank market (using equation (5) and employing the implicit function theorem reveals that $\partial K^{opt}_{i,0\le K_i\le R}(\cdot)/\partial R < 1$). A decreasing supply and an increasing demand obviously lead to a rising rate e.

The effect of transaction costs on the interbank market rate is ambiguous.[18] Rising transaction costs in the interbank market imply increasing marginal costs of the supplying banks. However, rising transaction costs do also lead to a decrease in

[17] The expression in brackets in equation (13) is positive: Since $0 < zh''/(f''+zh'') < 1$ implies $\int_0^{q_a^*} zh''g(q_i)/(f''+zh'')\,dqi < G(q_a^*)$, $\int_{q_b^*}^{q_c^*} zh''g(q_i)/(f''+zh'')\,dqi < G(q_c^*) - G(q_b^*) < 1 - G(q_b^*)$ and therefore $\partial e^*/\partial R > 0$.

[18] The first integral in equation (14) is positive since $h' < 0$ whereas the second integral is negative since here $h' > 0$.

demand since covering liquidity needs at the central banks becomes more favourable. It depends on the form of the cost functions $f(K_i)$ and $h(B_i)$ which effect outweighs and thus whether there is a decrease or increase in e.

The main findings of this section are summarized by the following result:

Result: *If the opportunity cost of collateral, which banks need to hold to obtain funds from the central bank, differ between banks, an interbank market will emerge. Banks with relatively low opportunity costs will act as an intermediary between the central bank and banks with higher costs. The interbank market rate will be higher than the central bank rate, with the difference being determined by total liquidity needs of the banking sector, average credit risk in the interbank market, transaction costs, the opportunity cost of holding collateral, and the distribution of the latter across banks.*

Illustration: Quadratic Cost Functions and Uniform Distribution

In order to illustrate the result of this paper graphically, we postulate the cost functions to be quadratic:

$$Q_i = q_i K_i + \frac{s}{2}(K_i)^2 \tag{15}$$

and

$$Z_i = \frac{z}{2} B_i^2 \tag{16}$$

with the parameters $s, z < 0$. Furthermore, we assume a uniform distribution of q_i, with $g(q_i) = 1$. In this case, we can draw figure 2, reflecting the equilibrium in the interbank market.

In panel (b), the upward sloping curves represent marginal cost of borrowing from the central bank given by

$$MC_{CB} = l + q_i + sK_i \tag{17}$$

for different levels of marginal opportunity costs captured by q_i. Since there is a continuum of banks differing in q_i, there is a continuum of marginal cost curves between l and $(l + q_{max})$. The solid downward sloping curve shows marginal costs of borrowing in the interbank market which are

$$MC_{IB} = e + zR - zK_i, \tag{18}$$

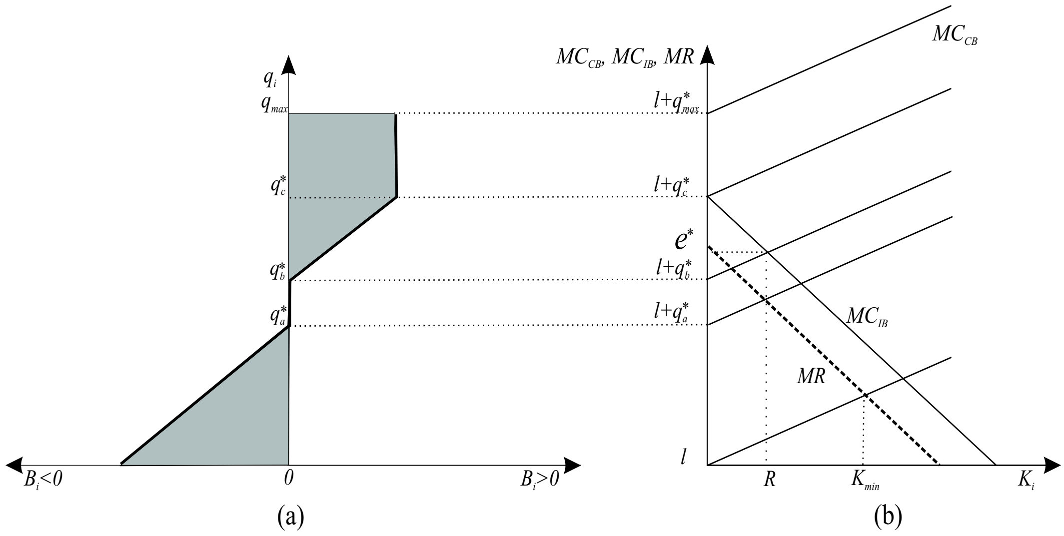

Figure 2: Interbank Market Equilibrium

whereas the dotted curve represents marginal revenue of placing liquidity in that market given by

$$MR = ep + zR - zK_i. \tag{19}$$

These two curves are the same for each bank.

For banks with $q_i > q_c^*$ marginal cost of borrowing from the central bank are always higher than of borrowing in the interbank market. Consequently, in this case $K_i^{opt} = 0$. For banks with $q_c^* > q_i > q_b^*$, $0 < K_i^{opt} < R$, i.e. they partially cover their liquidity needs at the central bank and in the interbank market. The bank-specific amount of central bank credit K_i^{opt} is determined by the intersection of the respective marginal cost curves. For banks with $q_a^* \leq q_i \leq q_b^*$, $K_i^{opt} = R$, i.e. these institutions cover exactly their own liquidity needs at the central bank, whereas credit institutions with $q_i < q_a^*$ borrow more reserves, i.e. $K_i^{opt} \geq R$, to place the excess liquidity in the interbank market. The bank-specific amount of credit is determined by the intersection of the respective upward sloping marginal cost curve and the downward sloping marginal revenue curve.

The interbank rate e is determined by the intersection of the specific marginal cost curve $(q_b^* + l + sK_i^{opt})$ and the interbank marginal cost curve. (At this intersection, $K_i^{opt} = R$. Replacing K_i^{opt} by R in equation (18) reveals this result.)

Panel (a) represents aggregate demand and supply of liquidity in the interbank market, assuming a uniform distribution of q_i. The shaded area to the left of the vertical q_i-axis represents aggregate supply, the respective area to the right aggregate demand. In equilibrium, both areas have to be of the same size. If at a rate e aggregate supply is smaller than aggregate demand, for example, e will increase, leading to an increase in q_a, q_b, and q_c, until both areas are of the same size.

4 Summary

For the conduct of monetary policy it is important to know the functioning of the interbank money market and the determinants of the interbank money market rate. Developing a simple interbank money market model with a heterogenous banking sector we show that besides for balancing daily liquidity fluctuations or for speculative purposes, banks enter the interbank money market because they differ in marginal cost of borrowing funds from the central bank. These cost differences imply that banks with relatively low marginal costs act as intermediaries between the central bank and credit institutions with relatively high marginal costs. This results in a positive spread between the interbank market and central bank rate. The determinants of this spread are: transaction costs and credit risk in the interbank market, total liquidity needs of the banking sector, collateral's opportunity costs, and the distribution of the latter across banks.

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Appendix: Testing Eonia and Euribor Spreads for Different Samples

Tables 2 to 4 report one-sided tests of the null hypothesis of a non-positive spread against the alternative of a positive spread using different samples.

	Eonia			1-week Euribor			2-week Euribor
	FT	VT	F+V	FT	VT	F+V	VT
Mean	0.095	0.067	0.076	0.134	0.030	0.064	0.021
(t-stat)	(14.958)	(24.739)	(27.432)	(16.569)	(17.657)	(19.466)	(8.822)
[p-value]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
N	293	627	920	293	620	913	369

Table 2: Tests using daily data, special events exluded. The interest rate data in the first block correspond to the Eonia spread while the second and third block show the one-week and two-week Euribor spread using daily data respectively. Tests are reported for the fixed rate tender period (FT), the variable rate tender period (VT) and for both periods combined (F+V). Two-week Euribor data is only available for part of the variable rate tender period. Each column reports the sample mean, its t-statistic, the p-value of the one-sided test of the null hypothesis mean ≤ 0 against the alternative mean > 0 and the sample size N. During the variable rate tender period the Eonia is compared with the minimum bid rate at the MROs while the Euribor rates are compared with the weighted average MRO rate. Special episodes are excluded for all spreads.

	Eonia			1-week Euribor			2-week Euribor
	FT	VT	F+V	FT	VT	F+V	VT
Mean	0.083	0.082	0.082	0.122	0.034	0.062	0.024
(t-stat)	(3.907)	(6.589)	(7.627)	(9.514)	(7.031)	(10.587)	(4.646)
[p-value]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
N	76	169	245	76	167	243	99

Table 3: Tests using days of settlement, special events included.

	Eonia			1-week Euribor			2-week Euribor
	FT	VT	F+V	FT	VT	F+V	VT
Mean	0.065	0.075	0.072	0.136	0.031	0.064	0.022
(t-stat)	(6.326)	(12.938)	(14.027)	(16.743)	(14.119)	(19.522)	(7.666)
[p-value]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
N	382	827	1209	382	816	1198	483

Table 4: Tests using daily data, special events included.